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Receding Horizon Control of Cocurrent First Order Hyperbolic Partial Differential Equation Systems

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Abstract−Cocurrent first order hyperbolic partial differential equations (PDE's) have finite impulse response (FIR) characteristics. A finite difference scheme that preserve these nice dynamic characteristics is recently developed [Choi, submitted]. Employing the resulting genuine FIR model, the design of receding horizon control is easier. In this paper, a receding horizon control scheme for cocurrent first order hyperbolic PDE systems is proposed using the FIR model and is elucidated with a tubular reactor example.

Key words: First Order Hyperbolic PDE's, Receding Horizon Control, Reaction-convection Processes

INTRODUCTION

Most of chemical processes are infinite dimensional systems because transport phenomena described by PDE's are often involved. When diffusive transport is negligible and convective transport is dominant, a process is described by a first order hyperbolic PDE. For instance, tubular reactors [Ray, 1981], fixed bed reactors [Stangeland and Foss, 1970] and pressure swinging adsorption [Ruthven and Sircar, 1994] are contained in such a category. Additional examples can be found in [Rhee et al., 1986]. On the other hand, when diffusive transport is not negligible, a process is described by second order parabolic PDE. Such processes include a fluidized bed reactor or a packed bed reactor.

For infinite dimensional systems, the design of fully distributed controllers such as optimal control [Wang, 1966; Lo, 1973; Balas, 1986] and their implementations are quite complicated. However, design and implementation of controllers for finite dimensional systems are very well developed. Hence, in most practices, the original infinite dimensional systems described by a PDE is spatially discretized into a finite dimensional approximate model and, then, a finite dimensional controller is designed and implemented. For diffusion dominant systems described by parabolic PDE's, there are infinite number of discrete modes among which only a finite number of modes are slow and all the rests are stable and fast [Balas, 1979; Friedman, 1976]. Hence, for such a system, a meaningful low dimensional approximation is possible through modal decomposition and a finite dimensional controller can be found [Christofides and Daoutidis, 1997]. However, for first order hyperbolic PDE's, all the modes have the same, or almost the same, energy and, thus, a low dimensional model through modal decomposition is not possible since a large number of modes are necessary for accurate approximation of the original system. Hence, traditionally, the optimal control approach was adopted for control of hyperbolic PDE systems, that leads to fully distributed infinite dimensional controllers [Wang, 1966; Lo, 1973; Balas, 1986]. However, such controllers suffer from the complicated design and implementation. To overcome this difficulty, a geometric control theory based design of infinite dimensional controller without resorting to the optimal control techniques was proposed in [Christofides and Daoutidis, 1996]. Recently, a digital regulation technique for first order hyperbolic PDE systems was also proposed exploiting iterative learning control in [Choi et al., 2001].

Receding horizon control, also called model predictive control, was quite successful in chemical process industry. Hence, the development of receding horizon control for first order hyperbolic PDE systems will be beneficial. Receding horizon control with blind spatial discretization is obvious. As mentioned above, a low dimensional model through modal decomposition is not possible for first order hyperbolic PDE systems. However, cocurrent first order hyperbolic PDE systems have nice characteristics that the real parts of all the eigenvalues are at negative infinity and thus have FIR characteristics. A finite difference scheme that preserves these good dynamic characteristics is recently proposed [Choi, submitted]. Exploiting this preserved FIR property with the finite difference scheme, design and analysis of receding horizon control is easier. In this paper, employing this FIR preserving scheme, we propose a receding horizon control that is suitable for cocurrent first order hyperbolic PDE systems. The proposed methodology is illustrated with a tubular reactor example.

FORMULATION

Consider a linear first order hyperbolic partial differential equation:

$$\frac{\partial q}{\partial t} = -A\frac{\partial q}{\partial z} + B(z)q + C(z)u \quad (1)$$

with the boundary condition

$$q(t, 0)=q_B$$

and the initial condition:

$$q(0, z)=q_0(z), \quad \forall z \in [0, L].$$

Such a system may be obtained from the linearization around a de-

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‡This paper is dedicated to Professor Hyun-Ku Rhee on the occasion of his retirement from Seoul National University.

sired steady state of a quasi-linear first order PDE system such as a reaction convection process:

$$\frac{\partial q}{\partial t}=-A\frac{\partial q}{\partial z}+F(q)q+G(q)u$$

or a nonlinear first order PDE system:

$$\frac{\partial q}{\partial t}=-A\frac{\partial q}{\partial z}+H(q,u).$$

From the hyperbolicity, the matrix A is simple and, by possibly changing coordinates, is assumed in the form

$$A=\begin{bmatrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & a_N \end{bmatrix}$$

where $a_1 \geq a_2 \geq \cdots \geq a_N > 0$. In other words, we consider the cocurrent case only. Hence, in scalar form, we have

$$\frac{\partial q_i}{\partial t}=-a_i\frac{\partial q_i}{\partial z}+B_i(z)q_i+C_i(z)u$$

where $B_i(z)$ and $C_i(z)$ are the *i*th row of B(z) and C(z), respectively.

Fully distributed measurement and actuation are hard to implement. Hence, we consider a typical chemical process control configuration of first order hyperbolic PDE systems where a finite number of control actuators and a finite number of point sensors are employed. Namely, as depicted in Fig. 1, a different control input is applied in each prespecified interval and states are measured at a finite number of locations by point sensors. From the configuration, the system is very unlikely to be controllable. Hence, the desired steady state may be outside the reachable region and thus may not be achievable with the above control actuators. In this case, the best state we can achieve with the above control configuration is the closest to the desired one in some sense. For simplicity, we assume that the linearization was achieved around this best achievable state.

We now summarize the characteristics of first order hyperbolic PDE systems that play an important role in later development. The most important property of first order hyperbolic PDE systems is the following that can be found from [Russel, 1978].

Theorem: Consider the linear first order hyperbolic PDE systems in (1) with u=0. Suppose, as assumed above, that

$a_1 \geq a_2 \geq \cdots \geq a_N > 0.$

Then, the eigenvalues of the operator

$$\mathcal{L}q=-A\frac{\partial q}{\partial z}+B(z)q$$

are in the form

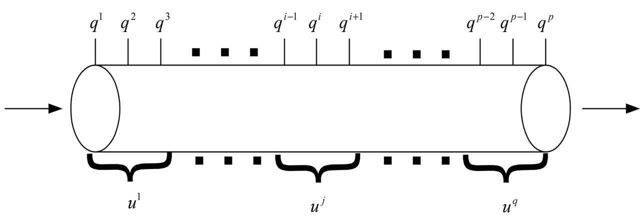

Fig. 1. Sensors and actuators.

$\lambda_n=-\infty+n\pi j.$

This mathematical theorem represents key characteristics of linear first order hyperbolic PDE systems. Firstly, this theorem says that all the eigenvalues with different imaginary parts (frequency mode) have the same, or almost the same, energy due to the infinite real part. Therefore, no one of them is dominant over the others and, thus, a low dimensional approximation with low frequency modes is difficult in general. Hence, a meaningful reduction can only be found with the information about initial conditions and inputs that set up and excite the each mode of frequency. Contrary to this negative effect, this theorem leads to a nice result that a linear first order hyperbolic PDE system has a finite impulse response property. It is well known that a linear finite dimensional continuous system can not have poles at $s=-\infty$. As a result, the best possible convergence is an exponential convergence and a finite step convergence is not possible. Moreover, placing the pole of the closed loop system at $s=-\infty$ is not physically possible since it requires an infinite instantaneous input. However, a discrete time system can achieve a finite step convergence when its poles are at the origin that corresponds to the poles at $s=-\infty$ of continuous case. However, as in the above theorem, the eigenvalues of $\mathcal{L}$ are on vertical line crossing the real axis at $s=-\infty$ and a finite time convergence is possible. This effect can be best understood with the method of characteristics for first order hyperbolic PDE systems. The method of characteristics adopts the characteristic line direction and spatial coordinate system instead of the temporal and spatial direction coordinate system. In the standard method of characteristics [Lapidus and Pinder, 1982; Rhee et al., 1986], however, the unit characteristic line direction vector is not used in the coordinate system and the discretization of the resulting equation can be confusing. Hence, the unit characteristic line direction vector is employed here. The characteristic line equation for q_i is

$$\frac{dz}{dt}=a_i$$

or

$z-a_it=\text{const.}$

The characteristic line direction vector must satisfy this constraint. Hence, the unit characteristic line direction vector is

$$\begin{bmatrix} \frac{a_i}{\sqrt{1+a_i^2}} \\ \frac{1}{\sqrt{1+a_i^2}} \end{bmatrix}.$$

Suppose τ and x represent the variables in the characteristic line direction and the spatial direction, respectively. Then, the coordinate transformation is

$$\begin{bmatrix} z \\ t \end{bmatrix}=\begin{bmatrix} 1 & \frac{a_i}{\sqrt{1+a_i^2}} \\ 0 & \frac{1}{\sqrt{1+a_i^2}} \end{bmatrix}\begin{bmatrix} x \\ \tau \end{bmatrix}.$$

Hence, in the characteristic line direction and spatial direction coor-

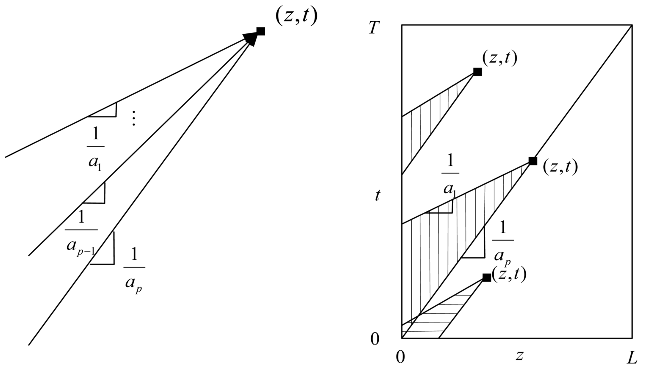

Fig. 2. Information flow for computation.

dinate,

$$\frac{\partial q_i}{\partial \tau}=\frac{\partial q_i}{\partial t}\frac{\partial t}{\partial \tau}+\frac{\partial q_i}{\partial z}\frac{\partial z}{\partial \tau}=\frac{1}{\sqrt{1+a_i^2}}B_i(z)q+\frac{1}{\sqrt{1+a_i^2}}C_i(z)u.$$

To illustrate the FIR property of cocurrent hyperbolic systems, we assume that u is not a feedback control input that can destroy the FIR property of the open loop system. As shown in the left figure of Fig. 2, the computation of $q_i(z, t)$ requires the information of q on the characteristic lines of q_i's in the third quadrant from (z, t). To compute each of the information, we again need the information on the characteristic lines emanating from it. Tracing the necessary information back this way, it is clear that the computation of q at (z, t) requires all the information in the cone formed by the characteristic lines of q_1 and q_N passing through (z, t). As shown in the right figure of Fig. 2, without feedback control, the computation of q on or above the diagonal does not require the initial conditions. On the other hand, the initial conditions are necessary to compute q below the diagonal. Hence, the region on which the initial conditions have influences is the triangle below the diagonal in the right figure of Fig. 2. To this end, the effects of the initial conditions die out in finite time.

Most of the control strategies are nowadays implemented with computer. Hence, we will consider a discrete time model predictive control. Therefore, we need to discretize the first order hyperbolic PDE in both time and space. To preserve the FIR property after discretization, the grid points in the upper initial condition independent triangle of Fig. 2 should be independent of the grid points below the triangle. However, all the known schemes such as upwind, Lax-Wendroff, Crank-Nicolson schemes [Lapidus and Pinder, 1982] utilize the points below the triangle due to temporal discretization and thus do not preserve the FIR property. Recently, a FIR property preserving finite difference scheme is developed for cocurrent first order hyperbolic PDE [Choi, submitted]. The key idea is to take difference along the characteristic line of a_N instead of the temporal differencing. However, for well defined states of the discretized system, rectangular grid points are necessary. To achieve these two goals simultaneously, the rectangular grid points are chosen so that a block formed by four adjacent grid points has the diagonal with slope $1/a_N$ as in Fig. 3. As shown in [Choi, submitted], the spatial differencing may need to be taken over several blocks for stability of the scheme. Indeed, the stability is guaranteed if $a_0=[(l+1)h]/k=(l+1)a_N\geq a_1$ where l is the number of the blocks over which

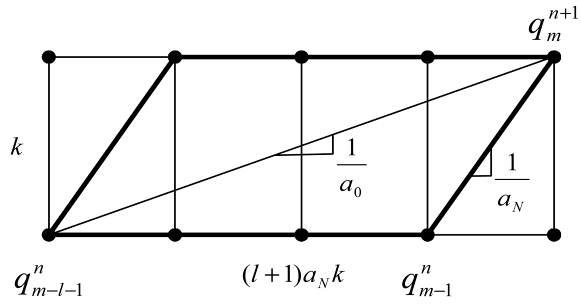

Fig. 3. Parallelogram for differencing.

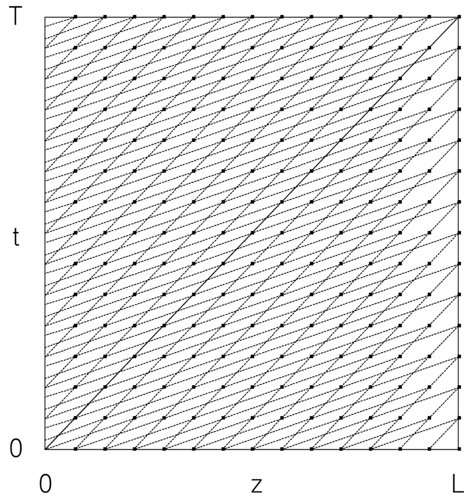

Fig. 4. Finite difference scheme.

the spatial differencing is taken. To this end, the overall picture of the gird points is shown in Fig. 4 where the solid dots represent grid points, the steep dotted lines have the slope $1/a_N$ and the gentle dotted lines have the slope $1/a_0$.

Now, for discretization, consider the coordinate consisting of the direction with the slope $1/a_N$ and the spatial direction. In this coordinate, we have

$$\frac{\partial q_i}{\partial \tau}=-\frac{(a_i-a_N)}{\sqrt{1+a_N^2}}\frac{\partial q_i}{\partial z}+\frac{1}{\sqrt{1+a_N^2}}B_i(z)q+\frac{1}{\sqrt{1+a_N^2}}C_i(z)u.$$

through the aforementioned coordinate transformation. Let k be the sampling interval in time $t=1/\sqrt{1+a_N^2}\,\tau$. Then, the sampling interval in spatial direction is $h=a_Nk$ and the length of the diagonal of a block is $\sqrt{1+a_N^2}$ k. Hence, the discretization over the parallelogram in Fig. 3 becomes

$$\frac{q_{i,m}^{n+1}-q_{i,m-1}^{n}}{k\sqrt{1+a_N^2}}=-\frac{(a_i-a_N)}{\sqrt{1+a_N^2}}\frac{q_{i,m-1}^{n}-q_{i,m-l-1}^{n}}{lh}+\frac{1}{\sqrt{1+a_N^2}}B_i((m-1)h)q_{m-1}^{n}+\frac{1}{\sqrt{1+a_N^2}}C_i((m-1)h)u_{m-1}^{n}$$

or

$$q_{i,m}^{n+1}=\left(1-\frac{a_i-a_N}{la_N}\right)q_{i,m-1}^{n}+kB_i((m-1)h)q_{m-1}^{n}+\frac{a_i-a_N}{la_N}q_{i,m-l-1}^{n}+kC_i((m-1)h)u_{m-1}^{n}.$$

As shown in [Choi, submitted], the discretization is consistent, nondissipative and nondispersive. Moreover, it is stable if $l\geq(a_1-a_N)/a_N$.

Near the left boundary ($m<l+1$), $m-l-1$ is negative and the above discretization is no longer valid if $a_i\neq a_N$. Now suppose $a_i\neq a_N$. The simplest implementation in this case is to use $q^n_{i,m-l-1}=q_{i,B}$ for $m-l-1<0$. Then, for $1<m<l+1$,

$$q^{n+1}_{i,m}=\left(1-\frac{a_i-a_N}{la_N}\right)q^n_{i,m-1}+kB_i((m-1)h)q^n_{m-1}+\frac{a_i-a_N}{la_N}q_{i,B}+kC_i((m-1)h)u^n_{m-1}$$

and

$$q^{n+1}_{i,1}=q_{i,B}+kB_i(0)q_B+kC_i(0)u^n_0.$$

However, when l is not small, this strategy can be very poor. A better but more intricate implementation in boundary can be found in [Choi, submitted].

Let L be the number of grid points contained in the zone of each control input. Then, it must hold that $u^n_{L*(j-1)+1}=u^n_{L*(j-1)+2}=\cdots=u^n_{L*j-1}$ where $j=1,\cdots,J$. Notice that J is the number of actuators. Then, we get the following discrete state space model through discretization:

$$x(p+1)=\mathcal{A}x(p)+\mathcal{B}v(p).$$

where

$$x(p)=\begin{bmatrix}q^p_1\\ \vdots\\ q^p_{JL}\end{bmatrix}\quad v(p)=\begin{bmatrix}u^p_1\\ \vdots\\ u^p_{(J-1)L+1}\end{bmatrix}$$

$$\mathcal{A}=\left[\begin{matrix}0 & 0 & 0\\ \mathrm{diag}\{1-\gamma_i\}+kB^1 & 0 & 0\\ 0 & \ddots & \ddots\\ 0 & \cdots & \mathrm{diag}\{1-\gamma_i\}+kB^{l-2}\\ \mathrm{diag}\{\gamma_i\} & 0 & \cdots\\ \vdots & \ddots & \ddots\\ 0 & \cdots & \mathrm{diag}\{\gamma_i\}\end{matrix}\right.$$

$$\left.\begin{matrix}0 & \cdots & 0\\ 0 & \cdots & 0\\ \ddots & \cdots & 0\\ 0 & \cdots & 0\\ \mathrm{diag}\{1-\gamma_i\}+kB^{l-1} & \cdots & 0\\ \ddots & \ddots & \vdots\\ 0 & \mathrm{diag}\{1-\gamma_i\}+kB^{JL-1} & 0\end{matrix}\right]$$

$$\mathcal{B}=\begin{bmatrix}kC^1 & 0 & \cdots & 0\\ \vdots & 0 & \cdots & 0\\ kC^L & 0 & \cdots & 0\\ 0 & kC^{L+1} & \cdots & 0\\ 0 & \vdots & \cdots & 0\\ 0 & kC^{2L} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & \cdots & 0 & kC^{(J-1)L+1}\\ 0 & \cdots & 0 & \vdots\\ 0 & \cdots & 0 & kC^{JL}\end{bmatrix}$$

$$\gamma_i=\frac{a_i-a_N}{la_N},\ \ B^r=B((r-1)h),\ \ C^r=C((r-1)h).$$

Notice that $\mathcal{A}$ is a nilpotent matrix and, thus, the system is an FIR system.

With this FIR model, any standard receding horizon control [Kwon and Pearson, 1977; Rawlings and Muske, 1993] or any model predictive control [Cutler and Ramaker, 1980] can be applied. From the FIR property, the receding horizon control with infinite horizon [Rawlings and Muske, 1993] does not have any advantage over the one with zero terminal state constraints [Kwon and Pearson, 1977] since the state converges to zero in a finite time in both cases. The DMC [Cutler and Ramaker, 1980] was designed with an FIR approximation of discretized infinite impulse response (IIR) plant model. But, for cocurrent first order hyperbolic PDE systems, the FIR plant model derived above is not an approximation of an IIR model and is even more accurate than the IIR model. With this FIR model, the design of receding horizon control is easier. For instance, the choice of prediction horizon is straightforward since the infinite prediction horizon can be reduced to a finite one.

Instead of the standard control schemes mentioned above, we will consider a slightly modified receding horizon control scheme where control actions are considered only over the initial condition dependent zone. Namely, nonzero control will be considered only in the smallest region covering the initial condition dependent zone (below the bold lines in Fig. 5). Then, the receding horizon control problem becomes

$$\min J(x(p))=\sum_{j=0}^{2JL-1}x(p+j+1|p)^TRx(p+j+1|p)$$

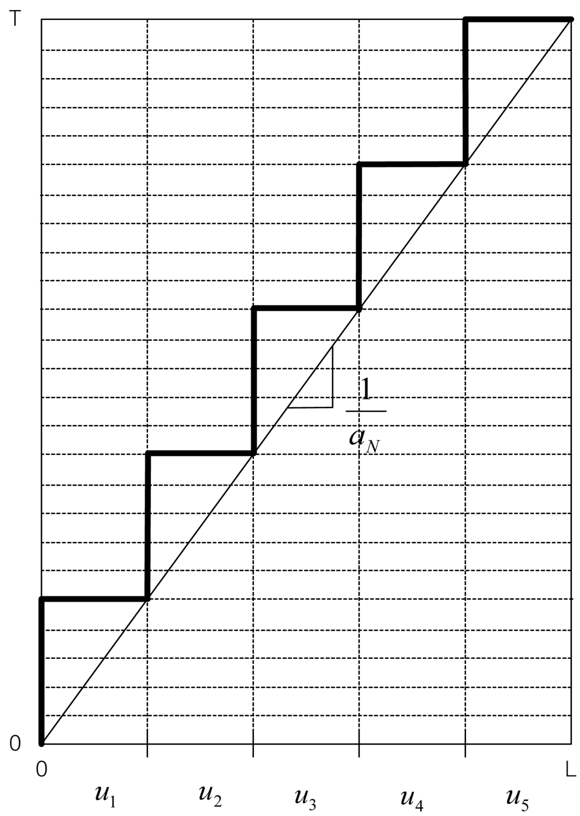

Fig. 5. Nonzero control zone.

$$+\sum_{j=0}^{2JL-1} u(p+j|p)^T S u(p+j|p)$$

subject to

$$x(p+j+1|p)=\mathcal{A}x(p+j|p)+\mathcal{B}u(p+j|p)$$
$$Gu(p+j|p)=0$$

where G is the matrix with 0 or 1 elements so that the control inputs in the region above the bold line in Fig. 5 become zero. Notice that the prediction horizon doesn't have to be greater than 2JL−1 from the FIR property since the states are zero after 2JL−1 time steps.

By decomposing the problem into different regions with different number of nonzero control inputs, the above receding horizon control problem can be rewritten as

$$\min J(x(p))=\sum_{s=1}^{J}\left\{\sum_{j=0}^{L-1} x_s(p+j+1|p)^T R x_s(p+j+1|p) + \sum_{j=0}^{L-1} u_s(p+j|p)^T S u_s(p+j|p)\right\} + \sum_{j=0}^{JL-1} x_{J+1}(p+j+1|p)^T R x_{J+1}(p+j+1|p)$$

subject to

$$x_s(p+j+1|p)=\mathcal{A}x_s(p+j|p)+\mathcal{B}_s u_s(p+j|p)$$
$$x_{s+1}(p|p)=x_s(p+L|p)$$

where

$$\mathcal{B}_1=\mathcal{B}$$

$$\mathcal{B}_s=\begin{bmatrix} kC^1 & 0 & \cdots & 0 \\ \vdots & 0 & \cdots & 0 \\ kC^L & 0 & \cdots & 0 \\ 0 & kC^{L+1} & \cdots & 0 \\ 0 & \vdots & \cdots & 0 \\ 0 & kC^{2L} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & kC^{(J-s)L+1} \\ 0 & \cdots & 0 & \vdots \\ 0 & \cdots & 0 & kC^{(J-s+1)L} \\ 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 \end{bmatrix},\quad s=2,\ldots,J$$

$$\mathcal{B}_{J+1}=0.$$

Notice that the last sum from j=0 to JL−1 can be replaced with the infinite sum. It is nothing more than

$$x_{J+1}(p|p)^T\left(\sum_{j=0}^{JL-1}(\mathcal{A}^T)^j R\mathcal{A}^j\right)x_{J+1}(p|p)=:x_{J+1}(p|p)^T R_{J+1} x_{J+1}(p|p)$$

or R_{J+1} can be obtained by the Lyapunov equation for the infinite sum

$$R_{J+1}=\mathcal{A}^T R_{J+1}\mathcal{A}+R.$$

Then, the solution of the problem can be obtained by the back tracking principle of dynamic programming. Namely, we need to successively solve the following standard linear quadratic optimal control problems whose solution can be found with Riccati equations [Lewis and Syrmos, 1995]:

$$\min\sum_{j=0}^{L-1} x_s(p+j+1|p)^T R x_s(p+j+1|p) + \sum_{j=0}^{L-1} u_s(p+j|p)^T S u_s(p+j|p) + x_s(p+L|p)^T R_s x_s(p+L|p)$$

subject to

$$x_s(p+j+1|p)=\mathcal{A}x_s(p+j|p)+\mathcal{B}_s u_s(p+j|p).$$

Notice that $x_s(p+L|p)^T R_s x_s(p+L|p)$ is the optimal cost $x_{s+1}(p|p)^T R_s x_{s+1}(p|p)$ of the (s+1)th problem.

Finally, the proof of the stability of the proposed control strategy can be established similarly to those in [Kwon and Pearson, 1977], [Rawlings and Muske, 1993] and [Choi and Kwon, 2003].

APPLICATION TO NONISOTHERMAL TUBULAR REACTOR

Consider the nonisothermal tubular reactor that is a reaction convection process. We assume a first order endothermic reaction takes place in the reactor:

$$A \rightarrow B$$

and the associated reaction kinetics follows the Arrhenius Law:

$$-\left(\frac{dC_A}{dt}\right)_{rxn}=k_0 e^{-\frac{E}{RT}} C_A$$

where C_A is the concentration of species A; T the reactor temperature; k_0 the pre-exponential constant; E the activation energy; R the gas constant. We adopt the following standard assumptions on the ideal tubular reactor:

- Perfect radial mixing takes place
- Diffusion is negligible
- Densities and heat capacities for A and B are the same and constant

Under these assumptions, the species balance for A and energy balance become

$$\frac{\partial C_A}{\partial t}=-v\frac{\partial C_A}{\partial z}-k_0 e^{-\frac{E}{RT}} C_A$$
$$\frac{\partial T}{\partial t}=-v\frac{\partial T}{\partial z}-\frac{\Delta H_r}{\rho c_p}k_0 e^{-\frac{E}{RT}} C_A+\frac{U}{\rho c_p V}(T_j-T)$$

with the boundary conditions

$$C_A(0,t)=C_A^0,\quad T(0,t)=T^0$$

and initial conditions

$$C_A(z,0)=C_{A0}(z),\quad T(z,0)=T_0(z)$$

where v is the velocity of the flow; ΔH_r the heat of reaction; ρ the density; c_p the heat capacity; T_j the jacket temperature, U the heat transfer coefficient; V the volume of reactor. The length L of the reactor is assumed 1 m. Notice that these are quasi-linear hyperbolic PDE's. The process parameters are listed in Table 1.

The desired steady state profile is assumed to be the one when

Table 1. Process parameters

Process parameter	Value
v (m/min)	1
E (cal/mol)	2.0×10^{4}
R (cal/mol·K)	1.987
ρ (kg/lt)	0.09
c_p (cal/kg·K)	700.231
k_0 (1/min)	5×10^{12}
U_w (cal/min·K)	2000.0
ΔH (cal/mol)	548.0001
V (lt)	10
L (m)	1

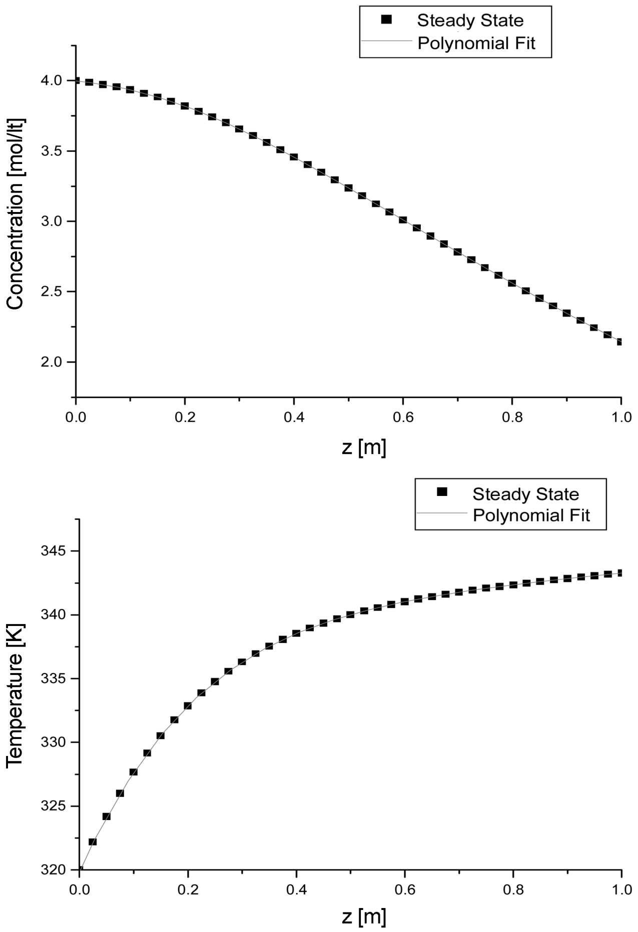

Fig. 6. Steady state profiles.

the jacket temperature is constant at 350 °K. It is depicted in Fig. 6. For the application of the control strategy proposed in this paper, we need linear hyperbolic PDE's. Hence we linearize the quasi-linear hyperbolic PDE's around the desired steady state. Since the exact solution of desired steady state is difficult to find, we find an analytic expression of the desired steady state through the regression with the 8th order polynomial (see Fig. 6) and use it for linearization. It is

$$C_{Ass}(z)=4.00005-0.44522z-1.72573z^2-5.06454z^3+12.70154z^4-8.70048z^5-1.92157z^6+5.06086z^7-1.76172z^8,$$

$$T_{ss}(z)=320.00048+91.32149z-159.62909z^2+122.33974z^3+23.97147z^4-115.3329z^5+76.04642z^6-13.00251z^7-2.43448z^8.$$

Since the shape of the desired steady state is simple, the fitting relative error with the 8th order polynomial was less than 10^{-4}. Through linearization, we have

$$\frac{\partial x_1}{\partial t}=-v\frac{\partial x_1}{\partial z}-k_0e^{-\frac{E}{RT_{ss}(z)}}x_1-k_0\frac{E}{RT_{ss}^2(z)}e^{-\frac{E}{RT_{ss}(z)}}C_{Ass}(z)x_2$$

$$\frac{\partial x_2}{\partial t}=-v\frac{\partial x_2}{\partial z}-\frac{\Delta H_r}{\rho c_p}k_0e^{-\frac{E}{RT_{ss}(z)}}x_1-\left(\frac{(\Delta H_r)}{\rho c_p}k_0\frac{E}{RT_{ss}^2(z)}e^{-\frac{E}{RT_{ss}(z)}}C_{Ass}(z)+\frac{U}{\rho c_p V}\right)x_2+\frac{U}{\rho c_p V}u$$

with the boundary conditions

$x_1(0, t)=0$, $x_2(0, t)=0$

and initial conditions

$x_1(z, 0)=x_{10}(z)$, $x_2(z, 0)=x_{20}(z)$

where

$x_1(t, z)=C_A(t, z)-C_{Ass}(z)$, $x_2(t, z)=T(t, z)-T_{ss}(z)$, $u(t, z)=T_j(t, z)-350$.

The discretization of this model using the technique proposed in the previous section corresponds to the one along the characteristic line. Along the characteristic line, we have the following ODE's along the characteristic line.

$$\frac{dx_1}{dt}=-k_0e^{-\frac{E}{RT_{ss}(vt)}}x_1-k_0\frac{E}{RT_{ss}^2(vt)}e^{-\frac{E}{RT_{ss}(vt)}}C_{Ass}(vt)x_2$$

$$\frac{dx_2}{dt}=-\frac{\Delta H_r}{\rho c_p}k_0e^{-\frac{E}{RT_{ss}(vt)}}x_1-\left(\frac{(\Delta H_r)}{\rho c_p}k_0\frac{E}{RT_{ss}^2(vt)}e^{-\frac{E}{RT_{ss}(vt)}}C_{Ass}(vt)+\frac{U}{\rho c_p V}\right)x_2+\frac{U}{\rho c_p V}u$$

These ODE's are discretized with the sampling time Δt=0.025 min.

Now we are ready to apply the control strategy proposed in this paper. For this, we assume the reactor is divided into five different zones with the same length and each zone is heated by a separate heating jacket. Moreover we assume the temperature and the concentration are measured at every discretized point by point sensors.

The weighting matrices associated with the receding horizon control are as follows:

$$R=\mathrm{diag}\begin{bmatrix}280 & 0\\ 0 & 0.4\end{bmatrix},\quad S=\mathrm{diag}\{0.1\}.$$

The simulation of the closed loop system starting from a non-steady state trajectory has been carried out. The simulation results are shown in Figs. 7, 8 and 9. The trajectories converge to the desired ones.

CONCLUSION

In this paper, recently developed FIR property preserving finite difference discretization for cocurrent hyperbolic PDE systems [Choi, submitted] is employed to proposed a model predictive control strat-

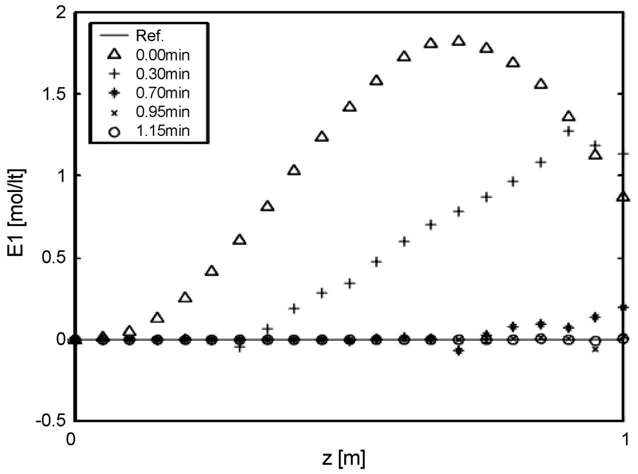

Fig. 7. Convergence of C_A.

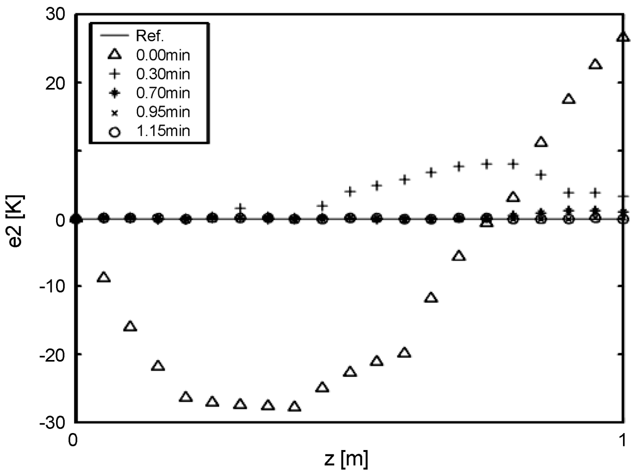

Fig. 8. Convergence of T.

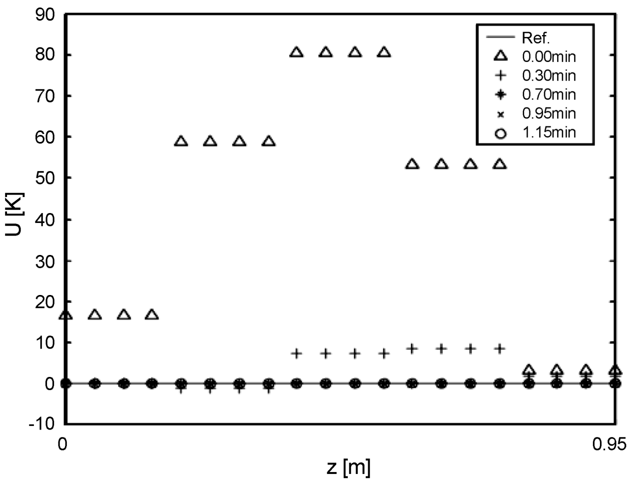

Fig. 9. Convergence of u.

egy with the resulting FIR model. Thanks to the genuine FIR property, the power of the proposed and other receding horizon control strategies can be exploited with relative ease. The efficiency of the proposed receding horizon control strategy is illustrated with a tubular reactor example.

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